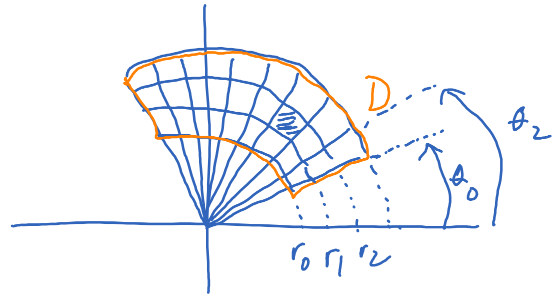
Integral over polar rectangle

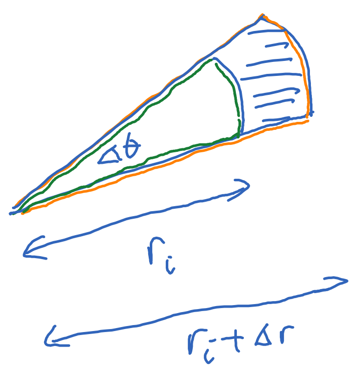
Wednesday, March 3, 2021 4:35 PM

$a \le r \le b$

$c \le \theta \le d$



$$\iint_D f(r,\theta)\,dA = \lim \sum_{ij} f(r_i,\theta_j)\,\Delta A_{ij}$$



Area of the red region is

$$\frac{1}{2}(r_i+\Delta r)^2\Delta\theta.$$

Area of the green region is

$$\frac{1}{2}r_i^2\Delta\theta$$

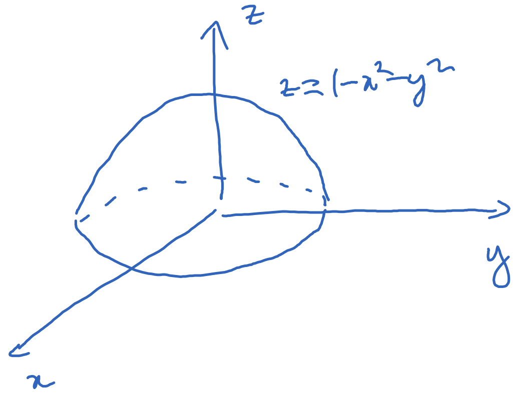
$$\Delta A_{ij} = \frac{1}{2}(r_i+\Delta r)^2\Delta\theta - \frac{1}{2}r_i^2\Delta\theta$$

$$= r_i\Delta r\Delta\theta + \frac{1}{2}r_i(\Delta r)^2\Delta\theta.$$

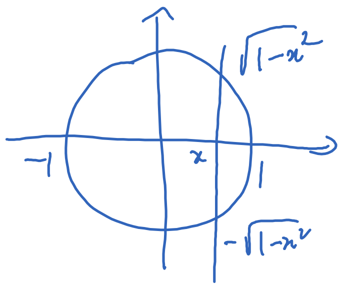
$$\approx r_i\Delta r\Delta\theta$$

$$\iint_D f(r,\theta)\,dA = \lim \sum f(r_i,\theta_j)\,r_i\Delta r\Delta\theta = \int_c^d\int_a^b f(r,\theta)\,r\,dr\,d\theta.$$

Ex :



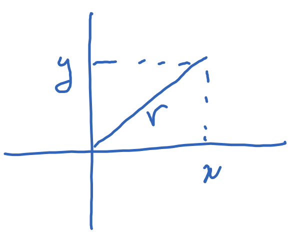
$$\text{Volume} = \iint_D (1 - x^2 - y^2)\, dA$$



$$\text{Volume} = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (1 - x^2 - y^2)\, dy\, dx$$

D is a polar rectangle : $0 \le r \le 1$, $0 \le \theta \le 2\pi$.

$x^2 + y^2 = r^2$



$$\text{volume} = \iint_D (1 - r^2)\, dA = \int_0^{2\pi} \int_0^1 (1 - r^2)\, r\, dr\, d\theta.$$